Calc BC §9.5 Pg 523 # 23-28, 35, 38, 42, 43,

(23) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2} = \frac{2}{1} - \frac{3}{4} + \frac{4}{9} - \frac{5}{16} + \ldots$ converges by A.S.T.

and $\sum_{n=1}^{\infty} \left| (-1)^{n+1} \frac{1+n}{n^2} \right| = \sum_{n=1}^{\infty} \frac{1+n}{n^2}$ this diverges by D.C.T

comparing to $\frac{1+n}{n^2} \geq \frac{1}{n}$ and $\frac{1}{n}$ diverges

$\therefore$ converges conditionally ; bound $= \frac{101}{100^2} = 0.0101$

(24) $\sum_{n=1}^{\infty} (-1)^{n+1} \left(\frac{1}{10}\right)^n = \frac{1}{10} - \frac{1}{100} + \frac{1}{1000} - \ldots$ converges by A.S.T.

$\sum_{n=1}^{\infty} \left(\frac{1}{10}\right)^n$ is geometric and converges

$\therefore$ converges absolutely ; bound $= \left(\frac{1}{10}\right)^{100}$

(25) $\sum_{n=2}^{\infty} (-1)^{n+1} \frac{1}{n \cdot \ln n} = -\frac{1}{2 \cdot \ln 2} + \frac{1}{3 \cdot \ln 3} - \ldots$ converges by A.S.T.

$\sum_{n=2}^{\infty} \frac{1}{n \cdot \ln n}$ diverges by integral test

$$\int_2^{\infty} \frac{1}{x \cdot \ln x} dx = \lim_{b \to \infty} \int_2^b \frac{1}{x \cdot \ln x} dx \qquad u = \ln x, \quad du = \frac{1}{x} dx$$

$$= \lim_{b \to \infty} \int_2^b \frac{1}{u} du$$

$$= \lim_{b \to \infty} \Big[\ln |\ln u| \Big]_2^b = \infty$$

$\therefore$ converges conditionally bound $= \frac{1}{100 \cdot \ln(100)}$

(26) $\sum_{n=1}^{\infty} (-1)^n n^2 \left(\frac{2}{3}\right)^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)^2 \left(\frac{2}{3}\right)^{n+1}}{n^2 \left(\frac{2}{3}\right)^n} = \frac{2}{3}$ Ratio Test

Converges absolutely bound $= 100^2 \left(\frac{2}{3}\right)^{100}$

(27) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!}$

$= \lim_{n\to\infty} \frac{n+1}{2} = \infty > 1$

$\therefore$ Diverges by Ratio Test

(28) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin n}{n^2}$

$\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^2}\right|$ converges by DCT using $\sum \frac{1}{n^2}$ which converges using P-series test.

(35) $\sum_{n=0}^{\infty} x^n$ geometric which converges for $|x| < 1$

a) $1 < x < 1$

b) $1 < x < 1$

c) none

(36) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$; $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{|3x-2|^{n+1}}{n+1} \cdot \frac{n}{|3x-2|^n}$

$= |3x-2|$

$|3x-2| < 1$

$-1 < 3x-2 < 1$

$\frac{1}{3} < x < 1$ → converges absolutely on $(\frac{1}{3}, 1)$

endpoints

$x = \frac{1}{3}$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ converges A.S.T

$x = 1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges

interval $[\frac{1}{3}, 1)$

a) $[\frac{1}{3}, 1)$

b) $(\frac{1}{3}, 1)$

c) At $x = \frac{1}{3}$

(42) $\sum_{n=0}^{\infty} \frac{x^{2n+1}}{n!}$: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x^{2n+3}|}{(n+1)!} \cdot \frac{n!}{|x^{2n+1}|}$

$= \lim_{n\to\infty} \frac{x^2}{n+1}$

$= 0$

$L < 1$, converges absolutely $\forall x$.

(43) $\sum_{n=0}^{\infty} \frac{n(x+3)^n}{5^n}$: $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)|x+3|^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n|x+3|^n}$

$= \lim_{n\to\infty} \left(\frac{n+1}{5n}\right) |x+3|$

$= \frac{|x+3|}{5} \Rightarrow \frac{|x+3|}{5} < 1$

$|x+3| < 5$

$-5 < x+3 < 5$

$-8 < x < 2$

this is where converges absolutely

endpoints $x=-8$: $\sum_{n=0}^{\infty} \frac{n(-5)^n}{5^n} = \sum_{n=0}^{\infty} n(-1)^n$

$= 0 - 1 + 2 - 3 + 4 - 5 \ldots$

$S_0 = 0$ $\quad S_4 = 2$

$S_1 = -1$ $\quad S_5 = -3$

$S_2 = 1$

$S_3 = -2$ $\quad$ diverges

$x = 2$: $\sum_{n=0}^{\infty} \frac{n(5^n)}{5^n} = \sum_{n=0}^{\infty} n$ diverges by n^{th} term

$(-8, 2)$